On the high derivative fermionic operator and trace anomaly

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ABSTRACT

We construct a new example of the high derivative four-dimensional conformal operator. This operator acts on fermions, and its contribution to the trace anomaly has opposite sign, as compared to conventional scalars, spinors and vectors. Possible generalizations and applications are discussed.

1 Introduction

The local conformal symmetry plays great role in gravitational physics. But, the most important is that, due to the quantum effects of the matter fields, conformal symmetry is violated by the trace anomaly [1]. This anomaly is relevant for the applications of quantum field theory on curved space-time [2, 3]. In particular, it is in the heart of such achievement as the first inflationary cosmological model of Starobinsky [4] and the semiclassical approach to the derivation of the Hawking radiation from the black holes [5]. Recently, there was a considerable interest to study the general properties of the anomaly through the anomaly induced effective action [6, 7] (see also [8] for the consequent discussions). In particular, this effective action has been used to obtain systematic classification of the black hole vacuum states [9] and for the more detailed analysis of the Starobinsky model [10]. One has to notice, that the anomaly-induced action is defined with accuracy to an arbitrary conformal invariant functional [11]. Since, in general, there is no regular way to derive the conformal part of the effective action, it is useful to have various versions of the conformal invariant actions, so that one could apply them to mimic the unknown conformal functional [9]. We remark, that the form of some conformal invariants constructed from curvature tensor has been already discussed my mathematicians (see, e.g. [12]).

The anomaly induced gravitational action is a direct $4d$ analog of the Polyakov action in $2d$. Since the quantum analysis of the Polyakov theory was led to the numerous interesting advances

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(starting from [13]), it is quite natural the emergence of the idea to use the $4d$ induced action of [6, 7] to construct the quantum theory of gravity [14, 15] and to generalize the c-theorem from $2d$ for the $4d$ space-time [16]. With respect to the renormalization group one can mention that the contribution of all matter fields to anomaly and to the renormalization group equations for the parameter of the vacuum action possess some amusing universality. If we write the vacuum action in the form

$$S_{vacuum} = \int d^4x\sqrt{-g}\,\Big\{a_1C^2 + a_2E + a_3\Box R\Big\}\,, \tag{1}$$

then all existing matter (non-gravitational) fields: scalars

$$S_0 = \int d^4x\sqrt{-g}\,\Big\{\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi + \frac{1}{12}\,R\phi^2\Big\} \tag{2}$$

spinors

$$S_{1/2} = \frac{i}{2}\int d^4x\sqrt{-g}\,\{\,\bar{\Psi}\gamma^\mu\nabla_\mu\Psi - \nabla_\mu\bar{\Psi}\,\gamma^\mu\Psi\,\} =$$

$$= i\int d^4x\sqrt{-g}\,\bar{\Psi}\gamma^\mu\nabla_\mu\Psi \;+\; \text{surface} \quad \text{term} \tag{3}$$

and vectors

$$S_1 = \int d^4x\sqrt{-g}\,\Big\{-\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}\Big\} \tag{4}$$

give positive contribution to the β-function of the parameter a_1 and negative contribution to the β-function of the parameter a_2 (see formulas (36) below). In relation to the beta-function of the a_2 parameter the situation remains the same in the framework of a supergravity theory. Besides (2), (3) and (4), conformal supergravity includes Weyl gravity (spin-2) and corresponding spin-3/2 fields. This universality has one important consequence: one can not cancel anomaly by choosing the appropriate number of the fields of different spins. Therefore, in this point one meets an important deviation between $2d$ and $4d$ cases, because in $2d$ the anomaly cancellation is indeed possible and this provides the existence of the critical dimension in string theory.

Taking into account the interest to the higher (than $2d$) dimensional conformal field theories, one can formulate two relevant questions: i) whether one can construct the conformal invariant theories distinct from the theories (2), (3) and (4)? ii) If this is possible, what would be the contribution to anomaly of these fields? In principle, it might happen that including some special amount of these new fields into the definition of the integration measure, one could cancel the anomaly. In this case these new conformal fields would violate universality of the renormalization group flow. Summing up, the second question can be formulated as: whether it is possible to maintain the conformal symmetry at quantum level by introducing some new fields?

Once the investigation of the second problem has been performed [17] and it was found that the cancellation is impossible for the fields which compose the conformal supergravity theory. As an example of the work done in direction i), one can indicate Ref. [18], where new tensor operators with local conformal symmetry have been constructed. On the other hand, it was noticed long ago [19, 6, 7] that there is the high derivative conformal operator

$$\Delta = \Box^2 + 2R^{\mu\nu}\,\nabla_\mu\nabla_\nu - \frac{2}{3}\,R\Box + \frac{1}{3}\,(\nabla_\mu\,R)\,\nabla^\mu\,. \tag{5}$$

Moreover, the contribution of the corresponding free scalar

$$S_4 \,=\, \int d^4x\sqrt{-g}\,\varphi\Delta\varphi \tag{6}$$

to the trace anomaly has opposite sign [6], as compared to the usual fields (2), (3) and (4)[3].

Why the existence of two different conformal scalars (2) and (6) is possible? The remarkable difference between two conformal scalars is the transformation law for the fields

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu}\, e^{2\sigma}\,, \qquad \phi \to \phi' = \phi\, e^{-\sigma}\,, \qquad \varphi \to \varphi' = \varphi\,. \tag{7}$$

The main purpose of the present article is to construct the spinor analog of the action (6). The form of the first integral in (3) is dictated by the requirement of action to be Hermitian. Furthermore, in the conventional spinor case (3) the conformal transformation has the form

$$\Psi \to \Psi' = \psi\, e^{-3\sigma/2}\,, \qquad \bar{\Psi} \to \bar{\Psi}' = \bar{\psi}\, e^{-3\sigma/2} \tag{8}$$

Therefore, in order to construct the high derivative conformal action, one has to try the Hermitian action

$$S_3 \,=\, \frac{i}{2}\int d^4x\sqrt{-g}\,\left\{\,\bar{\psi}\gamma^\mu\mathcal{D}_\mu\psi - \mathcal{D}_\mu\bar{\psi}\,\gamma^\mu\psi\,\right\}\,, \tag{9}$$

where $\mathcal{D}_\mu$ is some third derivative covariant operator, and postulate the following transformation law for the spinor ψ:

$$\psi \to \psi' = \psi\, e^{-\sigma/2}\,, \qquad \bar{\psi} \to \bar{\psi}' = \bar{\psi}\, e^{-\sigma/2}\,. \tag{10}$$

Indeed, these requirements guarantee the global conformal invariance with $\sigma = const$ [20]. However, the possibility to have local conformal invariance is not at all obvious and we are going to investigate it here.

The article is organized as follows. In section 2 we present the construction of the conformal operator $\mathcal{D}_\mu$, and in section 3 – the calculation of its contribution to the trace anomaly. In the last section we draw our conclusions and present some mathematical *conjecture* and some speculations about possible physical applications of this and other possible conformal operators.

2 The derivation of the conformal operator

The covariant derivative of the spinor is defined in a usual way

$$\nabla_\mu\psi = \partial_\mu\psi + \frac{i}{2}\,\omega^{ab}{}_\mu\Sigma_{ab}\psi\,, \quad \nabla_\mu\bar{\psi} = \partial_\mu\bar{\psi} - \frac{i}{2}\,\omega^{ab}{}_\mu\bar{\psi}\Sigma_{ab}\,,$$

where $\Sigma_{ab} = \frac{i}{2}\,(\gamma_a\gamma_b - \gamma_b\gamma_a)$. Consequently,

$$[\nabla_\mu\,,\nabla_\nu]\,\psi = \hat{\mathcal{R}}_{\mu\nu}\;\psi \;=\; \frac{1}{4}\,\gamma^\alpha\gamma^\beta R_{\alpha\beta\mu\nu}\,\psi\,.$$

[3]In [6] the contribution of (5) to the anomaly was taken with negative sign, for this was taken as the compensation of the integration over the auxiliary field.

Using dimensional reasons, one can fix the possible form of the operator $\mathcal{D}_\mu$ as

$$\mathcal{D}_\mu = \nabla_\mu \Box + k_1 \, R_{\mu\nu} \nabla^\nu + k_2 \, R \nabla_\mu \,, \tag{11}$$

where k_1 and k_2 are some unknown coefficients. Integrating by parts and omitting the surface terms, we get

$$S \,=\, i \int d^4x \sqrt{-g}\, \bar{\psi} \gamma^\mu \tilde{\mathcal{D}}_\mu \psi + \,... \tag{12}$$

where

$$\tilde{\mathcal{D}}_\mu = \frac{1}{2}(\nabla_\mu \Box + \Box \nabla_\mu) + k_1 R_{\mu\nu} \nabla^\nu + k_2 R \nabla_\mu + \left(\frac{k_1}{4} + \frac{k_2}{2} \right) (\nabla_\mu R) \,. \tag{13}$$

Making commutations of the covariant derivatives

$$\frac{1}{2}(\nabla_\mu \Box + \Box \nabla_\mu) = \nabla_\mu \Box + \frac{1}{2}[\Box \,, \nabla_\mu] \tag{14}$$

one can calculate

$$\begin{aligned} [\Box \,, \nabla_\mu] \psi \;&=\; [\nabla_\rho \,, \nabla_\mu] \nabla^\rho \psi + \nabla^\rho [\nabla_\rho \,, \nabla_\mu] \psi = \\ &=\; -\frac{1}{2} \gamma^\alpha \gamma^\beta R_{\alpha\beta\mu\rho} \nabla^\rho \psi + R_{\mu\rho} \nabla^\rho \psi - \frac{1}{4} \gamma^\alpha \gamma^\beta (\nabla^\rho R_{\alpha\beta\mu\rho}) \psi \,. \end{aligned} \tag{15}$$

Substituting (15) into (13), we obtain the useful form of the operator.

$$\tilde{\mathcal{D}}_\mu \,=\, \nabla_\mu \Box + a_1 R_{\mu\rho} \nabla^\rho + a_2 R \nabla_\mu + a_3 (\nabla_\mu R) \,, \tag{16}$$

where $a_1 = k_1$, $a_2 = k_2$ e $a_3 = \frac{a_1}{4} + \frac{a_2}{2} - \frac{1}{8}$ – the last is a condition of Hermiticity. Our purpose will be to find such a values of $a_{1,2,3}$, which provide both Hermiticity and conformal invariance of the action (9).

For the one-parameter Lie group, one can safely restrict the consideration by the infinitesimal version of the transformation

$$g_{\mu\nu} \to g^{'}_{\mu\nu} = (1+2\sigma) g_{\mu\nu} \,, \qquad \psi \to \psi^{'} = (1-\sigma/2)\psi \,, \qquad \bar{\psi} \to \bar{\psi}' = (1-\sigma/2)\bar{\psi} \,.$$

Then, disregarding the highest orders in σ, after some long algebra we arrive at the following transformations

$$\begin{aligned} (\bar{\psi}\gamma^\mu \nabla_\mu \Box \psi)^{'} \;&=\; -(\nabla_\mu \bar{\psi} \gamma^\mu \Box \psi)^{'} + \nabla^{'}_\mu (\bar{\psi}\gamma^\mu \Box \psi)^{'} = \\ &=\; -(1-4\sigma) \nabla_\mu \bar{\psi} \gamma^\mu \Box \psi - \nabla_\mu \sigma \bar{\psi} \gamma^\mu \Box \psi - \nabla_\mu \bar{\psi} \gamma^\mu \nabla_\nu \sigma \nabla^\nu \psi + \\ &+\; i \nabla_\mu \bar{\psi} \gamma^\mu \Sigma_{\rho\nu} \nabla^\nu \sigma \nabla^\rho \psi + \frac{1}{2} \nabla_\mu \bar{\psi} \gamma^\mu \Box \sigma \psi + \nabla^{'}_\mu (\bar{\psi}\gamma^\mu \Box \psi)^{'} \,, \end{aligned} \tag{17}$$

$$\begin{aligned} (\bar{\psi}\gamma^\mu R_{\mu\rho} \nabla^\rho \psi)^{'} = \qquad & (1-4\sigma) \bar{\psi} \gamma^\mu R_{\mu\rho} \nabla^\rho \psi - \frac{i}{2} \bar{\psi} \gamma^\mu R_\mu{}^\nu \Sigma_{\nu\rho} \nabla^\rho \sigma \psi - \\ -\; & \frac{1}{2} \bar{\psi} \gamma^\mu R_{\mu\nu} \nabla^\nu \sigma \psi - 2 \bar{\psi} \gamma^\mu \nabla_\mu \nabla_\nu \sigma \nabla^\nu \psi - \\ -\; & \bar{\psi} \gamma^\mu \Box \sigma \nabla_\mu \psi \,, \end{aligned} \tag{18}$$

$$(\bar{\psi}\gamma^\mu R\nabla_\mu\psi)' = (1-4\sigma)\bar{\psi}\gamma^\mu R\nabla_\mu\psi - 6\bar{\psi}\gamma^\mu\Box\sigma\nabla_\mu\psi + \bar{\psi}\gamma^\mu R\nabla_\mu\sigma\psi\,, \tag{19}$$

$$(\bar{\psi}\gamma^\mu\nabla_\mu R\psi)' = (1-4\sigma)\bar{\psi}\gamma^\mu\nabla_\mu R\psi - 2\bar{\psi}\gamma^\mu R\nabla_\mu\sigma\psi - 6\bar{\psi}\gamma^\mu\nabla_\mu\Box\sigma\psi\,. \tag{20}$$

Substituting these formulas into (12) with (16), we find that the conformal invariance

$$(\sqrt{-g}\,\bar{\psi}\gamma^\mu\tilde{\mathcal{D}}_\mu\psi)' = \sqrt{-g}\,\bar{\psi}\gamma^\mu\tilde{\mathcal{D}}_\mu\psi \tag{21}$$

holds for the unique choice of the Hermitian parameters

$$a_1 = 1\,, \qquad a_2 = -\frac{5}{12}\,, \qquad a_3 = -\frac{1}{12}\,. \tag{22}$$

3 One-loop divergences and anomaly

The one-loop effective action, for the free theory of field ψ can be presented in the form

$$\Gamma^{(1)} = -i\,\mathrm{Tr}\,ln(\gamma^\mu\tilde{\mathcal{D}}_\mu) = -\frac{i}{2}\,\mathrm{Tr}\,ln(\gamma^\mu\tilde{\mathcal{D}}_\mu\gamma^\nu\tilde{\mathcal{D}}_\nu)\,, \tag{23}$$

that can be further rewritten as

$$\gamma^\mu\tilde{\mathcal{D}}_\mu\gamma^\nu\tilde{\mathcal{D}}_\nu = \tilde{\mathcal{D}}_\mu\tilde{\mathcal{D}}^\mu + \frac{1}{2}\gamma^\mu\gamma^\nu\left[\tilde{\mathcal{D}}_\mu\,,\,\tilde{\mathcal{D}}_\nu\right]\,. \tag{24}$$

It proves useful to reduce the last expression to the form of the minimal six derivative operator

$$\gamma^\mu\tilde{\mathcal{D}}_\mu\gamma^\nu\tilde{\mathcal{D}}_\nu = \hat{H} = \hat{1}\,\Box^3 + \hat{V}^{\mu\nu}\nabla_\mu\nabla_\nu\Box + \hat{Q}^{\mu\nu\alpha}\nabla_\mu\nabla_\nu\nabla_\alpha + \hat{U}^{\mu\nu}\nabla_\mu\nabla_\nu + \hat{N}^\mu\nabla_\mu + \hat{P}\,. \tag{25}$$

By dimensional reasons, the $\hat{Q}^{\mu\nu\alpha}$, $\hat{N}^\mu$ and $\hat{P}$ - terms can not contribute to the divergences, and therefore have no interest for us. Indeed, this can be checked explicitly. The derivation of the divergences can be performed using the generalized Schwinger-DeWitt technique developed in [21]. Since all the steps in this calculus are quite similar to the ones presented in [21] for the four-derivative operator, we will not exhibit the details here. The general expression for the divergences has the form

$$-\frac{i}{2}\,\mathrm{Tr}\,ln\,\hat{H}\Big|_{div} = -\frac{\mu^{n-4}}{(4\pi)^2\,(n-4)}\int d^nx\sqrt{-g}\,\Big\{\frac{7}{120}\,R^2_{\mu\nu\alpha\beta} + \frac{1}{15}\,R^2_{\mu\nu} - \frac{1}{6}\,R^2 - \frac{2}{5}\,\Box R +$$

$$+\,\mathrm{tr}\,\Big[\frac{1}{2}\hat{V}^{\mu\nu}\hat{\mathcal{R}}_{\mu\nu} + \frac{1}{6}\,\hat{V}^{\mu\nu}\,(R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu}) + \frac{1}{4}\,g^{\mu\nu}\,\hat{U}_{\mu\nu} - \frac{1}{24}\,\hat{V}^{\mu\nu}\hat{V}_{\mu\nu} - \frac{1}{48}\,(\hat{V}^{\mu\nu}g_{\mu\nu})^2\Big]\Big\}\,, \tag{26}$$

and of course they do not depend on $\hat{Q}^{\mu\nu\alpha}$, $\hat{N}^\mu$ and $\hat{P}$ - terms.

Now we are in a position to calculate the divergent part of the one-loop effective action using (26). For the sake of generality we shall perform the calculations for arbitrary values of $a_{1,2,3}$, and will substitute (22) only afterwards. After some long calculus, disregarding the non-essential terms

with more than four derivatives of the external metric, we arrive at the relations

$$
\begin{aligned}
\gamma^\mu\gamma^\nu\left[\tilde{\mathcal{D}}_\mu\,,\,\tilde{\mathcal{D}}_\nu\right] &= \frac{1}{8}\gamma^\alpha\gamma^\beta\gamma^\mu\gamma^\nu R_{\alpha\beta\rho\lambda}R_{\mu\nu}{}^{\rho\lambda}\Box - 2\gamma^\mu\gamma^\nu R_{\nu\rho}\nabla_\mu\nabla^\rho\Box - \\
&- \frac{1}{2}\Box R\Box - \frac{1}{2}R\Box^2 + [\gamma^\mu,\gamma^\nu]R_{\nu\rho}\nabla_\mu\nabla^\rho\Box + \\
&+ a_1\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\left(-R_{\alpha\beta\sigma\rho}R^\sigma{}_{[\nu}\nabla_{\mu]}\nabla^\rho + \frac{1}{2}R_{\alpha\beta[\mu}{}^\sigma R_{\nu]\sigma}\Box\right) + \\
&+ a_2R^2\Box - 2a_2\gamma^\mu\gamma^\nu RR_{\nu\rho}\nabla_\mu\nabla^\rho + \,... \qquad (27) \\
\tilde{\mathcal{D}}_\mu\tilde{\mathcal{D}}^\mu &= \Box^3 + (2a_1+1)R_{\mu\nu}\nabla^\mu\nabla^\nu\Box - \frac{1}{16}\gamma^\alpha\gamma^\beta\gamma^\lambda\gamma^\sigma R_{\alpha\beta\mu\rho}R_{\lambda\sigma}{}^{\mu\rho}\Box + \\
&+ a_1\Box R^{\mu\nu}\nabla_\mu\nabla_\nu + 2a_1\nabla_\mu\nabla_\rho R^{\mu\sigma}\nabla^\rho\nabla_\sigma + \\
&+ (a_1+a_1^2)R^{\mu\sigma}R_{\sigma\rho}\nabla_\mu\nabla^\rho + (a_2+a_3)\Box R\Box + \\
&+ 2(a_2+a_3)\nabla_\mu\nabla_\nu R\nabla^\mu\nabla^\nu + (a_2+2a_1a_2)RR_{\mu\nu}\nabla^\mu\nabla^\nu + \\
&+ 2a_2R\Box^2 + a_2^2R^2\Box + \,...\,. \qquad (28)
\end{aligned}
$$

Then, the relevant blocks of (25) are

$$
\begin{aligned}
\hat{V}^{\mu\nu} &= 2a_1R^{\mu\nu} + (2a_2 - \frac{1}{4})Rg^{\mu\nu} \qquad (29) \\
\hat{U}^{\mu\nu} &= a_1\Box R^{\mu\nu} + 2a_1\nabla_\rho\nabla^\mu R^{\rho\nu} + (a_2+a_3-\frac{1}{4})g^{\mu\nu}\Box R + \\
&+ (a_1^2+a_1)R^\mu{}_\rho R^{\rho\nu} + 2(a_2+a_3)\nabla^\mu\nabla^\nu R + (2a_1a_2+a_2)RR^{\mu\nu} + \\
&+ (a_1^2 - \frac{a_2}{2})g^{\mu\nu}R^2 - \frac{1}{2}a_1\gamma^{[\mu}\gamma^{\rho]}\gamma^\alpha\gamma^\beta R_{\alpha\beta}{}^{\sigma\nu}R_{\sigma\rho} - \frac{a_1}{2}R^2_{\alpha\beta}g^{\mu\nu} - \\
&- a_2\gamma^\mu\gamma^\rho R_\rho{}^\nu R\,. \qquad (30)
\end{aligned}
$$

Using the formula (26), for the generic operator (16) we arrive at the following divergences

$$
\Gamma^{(1)}_{\rm div} = -\frac{1}{\varepsilon}\int d^4x\sqrt{-g}\,\left\{\,\alpha R^2_{\mu\nu\alpha\beta} + \beta R^2_{\mu\nu} + \gamma R^2 + \delta\Box R\,\right\}\,, \qquad (31)
$$

where

$$
\begin{aligned}
\alpha &= \frac{7}{120}\,, \qquad \beta = \frac{1}{3}a_1^2 - \frac{2}{3}a_1 + \frac{1}{15}\,, \qquad (32) \\
\gamma &= -\frac{1}{3}a_1^2 - 4a_2^2 - 2a_1a_2 - \frac{1}{6}a_1 - \frac{4}{3}a_2 - \frac{1}{8}\,, \\
\delta &= 2a_1 + 6a_2 + 6a_3 - \frac{7}{5}\,.
\end{aligned}
$$

Replacing the coefficients (22) corresponding to the Hermitian conformal operator, and using the basis of the square of the Weyl tensor, Gauss-Bonnet term

$$
C^2 = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta} = R^2_{\mu\nu\alpha\beta} - 2R^2_{\mu\nu} + \frac{1}{3}R^2\,, \qquad E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\mu\nu} + R^2
$$

and R^2, we arrive at the final result

$$
\Gamma^{(1)}_{\rm div} \,=\, -\,\frac{\mu^{(n-4)}}{\varepsilon}\int d^nx\sqrt{-g}\,\left\{\,-\frac{1}{60}C^2 + \frac{3}{40}E - \frac{12}{5}\Box R\,\right\}\,, \qquad (33)
$$

which is conformal invariant up to total derivatives. The cancellation of the non-conformal $\sqrt{-g}R^2$-term confirms the correctness of our calculations of both conformal operator and divergences.

The conformal anomaly is directly related to the divergences [1], so that we have, using (33):

$$< T^{\mu}{}_{\mu} >= -\frac{1}{2}\frac{1}{\sqrt{-g}}\frac{\delta \Delta S}{\delta \sigma} = -\frac{1}{(4\pi)^2}\,\left(\omega C^2 + bE + c\Box R\right)\,. \tag{34}$$

with

$$\omega = -\frac{1}{60}\,, \qquad\qquad b = +\frac{3}{40}\,, \qquad\qquad c = -\frac{12}{5} \tag{35}$$

Consider the cancellation of anomaly. In the four-dimensional space ($D = 4$), if one has only conventional scalar, spinor and vector fields, the cancellation of anomaly is impossible due to the fact that all these fields contribute to the coefficients ω and b (35) with the same signs. But, situation might change if we have some high derivative fields. For instance, some examples of finite and anomaly-free theory has been given in [22], where the IR and UV conformal fixed points of the renormalization group flow were established. Now, we shall see, that including the new conformal fields one can achieve the anomaly cancellation in a different way.

Imagine that we have a theory with N_0 real massless conformal invariant scalars (2), $N_{1/2}$ Dirac spinors (3) and N_1 massless vectors. In addition, the theory includes n_3 copies of the high derivative spinor (9) and n_4 copies of the high derivative scalar (6). Then the total expression for the anomaly is (34), with the following total coefficients:

$$\begin{aligned}
\omega_t &= \left(\frac{1}{120}\,N_0 + \frac{1}{20}\,N_{1/2} + \frac{1}{10}\,N_1\right) - \left(\frac{1}{15}\,n_4 + \frac{1}{60}\,n_3\right)\,,\\
b_t &= -\left(\frac{1}{360}\,N_0 + \frac{11}{360}\,N_{1/2} + \frac{31}{180}\,N_1\right) + \left(\frac{7}{90}\,n_4 + \frac{3}{40}\,n_3\right)\,,
\end{aligned}$$

$$c_t = \frac{N_0}{180} + \frac{N_{1/2}}{30} - \frac{N_1}{10} - \frac{2}{45}\,n_4 - \frac{12}{5}\,n_3\,. \tag{36}$$

In order to obtain the conditions of anomaly cancellation, one has to consider only the coefficients ω_t and b_t, because c_t can be always canceled by adding the local finite counterterm [1]

$$\Delta S_c \;=\; \frac{c_t}{12\,(4\pi)^2}\,\int d^4x\sqrt{-g}\,R^2\,. \tag{37}$$

Then, from (36) one arrives at the following solutions for the amount of the high derivative fields

$$n_3 = N_1 - \frac{1}{2}\,N_{1/2} - \frac{1}{8}\,N_0\,, \qquad\qquad n_4 = \frac{5}{4}\,N_1 + \frac{7}{8}\,N_{1/2} + \frac{5}{32}\,N_0\,. \tag{38}$$

Indeed, both n_3 and n_4 must be integers. One can see, that the cancellation of trace anomaly is, in principle, possible, but it puts some restrictions on the field composition $N_{0,1/2,1}$ of the matter theory. Namely: we need $N_0 + 4N_{1/2} \leq 8N_1$, N_0 to be multiple of 32, $N_{1/2}$ to be multiple of 8 and N_1 to be multiple of 4. The last conditions can be easily satisfied for some gauge groups. Then, in some theory with extended supersymmetry, if the one-loop anomaly is exact, the proper choice of the numbers n_3 and n_4 can provide the complete cancelation of anomaly.

4 Conclusions and speculations

We have constructed the 3-derivative spinor action which possesses local conformal invariance. Our solution is a generalization of the flat-space operators [20] with global scale invariance. The relation between the corresponding spinor and conventional massless Dirac spinor is similar to the one between fourth derivative scalar (6) and usual conformal scalar (2). One can formulate the following mathematical *conjecture*: those are only first representatives of the infinite family of the conformal invariant operators with even (for scalars) and odd (for spinors) number of derivatives in $D = 4$. The scalars and spinors corresponding to these operators transform according to their classical dimension. The generalization to the $D \neq 4$ is also possible (see [23] for the Δ operator). To check this *conjecture* would be an interesting mathematical problem.

The contributions of a new third derivative spinor to the trace anomaly has the sign opposite to the one of the usual scalars, spinors and vectors, and the same as for the high derivative scalar (6). One can guess that this sign distribution is related to the emergence and dominating contributions of the high derivative unphysical ghosts, which are always present in the spectrum of the high derivative operators. As an extension of our *conjecture*, one can suppose that the signs of the contributions of the (yet unknown) higher order conformal operators to the trace anomaly coefficients ω_t and b_t will alter, as in (36). Therefore, if there exists a SYM theory with extended supersymmetry, for which the one-loop anomaly is exact, one can define the integration measure, in curved space-time, in such a way that the $D = 4$ conformal symmetry is exact. This definition of the measure must include proper number of functional determinants of the operators like Δ and $\gamma^\mu \tilde{\mathcal{D}}_\mu$. It might happen, that the investigation of this hypothesis can shed some light on the supersymmetry breaking mechanism which may be, after all, related to the conformal anomaly.

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References

[1] M.J. Duff, Nucl. Phys. **125 B** (1977) 334.

[2] N.D. Birell and P.C.W. Davies, *Quantum fields in curved space* (Cambridge Univ. Press, Cambridge, 1982).

[3] M.J. Duff, Class. Quant. Grav. **11** (1994) 1387.

[4] A.A. Starobinski, Phys.Lett. **91B** (1980) 99.

[5] S.M. Christensen and S.A. Fulling, *Phys. Rev.* **D15** (1977) 2088.

[6] R.J. Riegert, Phys. Lett. **B 134** (1984) 56.

[7] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. **B 134** n.3,4 (1984) 187.

[8] S. Deser and A. Schwimmer, Phys.Lett. **309B** (1993) 279;
S. Deser, Phys.Lett. **479B** (2000) 315.

[9] R. Balbinot, A. Fabbri and I.L. Shapiro, Nucl.Phys. **B559** (1999) 301.

[10] J.C. Fabris, A.M. Pelinson, I.L. Shapiro, *On the gravitational waves on the background of anomaly-induced inflation.* Nucl. Phys. **B**, to be published.

[11] I.L. Buchbinder, S.D. Odintsov and I.L. Shapiro, Phys.Lett. **162B** (1985) 92.

[12] T.P. Branson, Comm. Partial Diff. Equations **7** (1983) 305; Comm. Math. Phys. **178** (1996) 301.

[13] V.G. Kniznik, A.M. Polyakov and A.B. Zamolodchikov, Mod. Phys. Lett. **3A** (1988) 819.

[14] S.D. Odintsov and I.L. Shapiro, Class. Quant. Gravity. **8** (1991) L57.

[15] I. Antoniadis and E. Mottola, Phys. Rev. **45D**, 2013 (1992).

[16] J.L. Cardy, Phys. Lett. **215B** (1988) 749;
I. Jack and H. Osborn, Nucl. Phys. **B343** (1990) 647;
A. Capelli, D. Friedan and J.I. Latorre, Nucl. Phys. **B352** (1991) 616;
D.Z. Freedman and H. Osborn, Phys. Lett. **432B** (1998) 353.

[17] E.S. Fradkin and A.A. Tseytlin, Phys.Repts. **119** (1985) 233.

[18] J. Erdmenger, Class. Quant. Grav. **14** (1997) 2061;
J. Erdmenger and H. Osborn, Class. Quant. Grav. **15** (1998) 273.

[19] S. Paneitz, A Quartic Conformally Covariant Differential Operator for Arbitrary Pseudo-Riemannian Manifolds, MIT preprint, 1983 (unpublished).

[20] H.P. Dürr and P. du T. van der Merwe, Il Nuovo Cim. **23 A,n.1** (1974) 1;
C.C. Chiang and H.P. Dürr, Il Nuovo Cim. **28 A,n.1** (1975) 89.

[21] A.O. Barvinsky and G.A. Vilkovisky, Phys. Repts. **119** (1985) 1.

[22] E.Elizalde, A.G.Jacksenaev, S.D.Odintsov and I.L.Shapiro, Class.Quant.Grav. **12** (1995) 1385.

[23] J.A. de Barros and I.L. Shapiro, Phys. Lett. **412B** (1997) 242.